International Journal of Solids and Structures 41 (2004) 2043–2064

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Stationary damage modelling of poroelastic contact

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Received 25 June 2002; received in revised form 20 August 2003

Abstract

This paper introduces the concept of *stationary damage* as a means for examining the mechanical behaviour of fluid saturated media that are susceptible to alteration in their elastic and fluid transport characteristics, while maintaining the essential poroelastic character of the medium. In this study, the effects of elastic damage and the attendant alterations in the hydraulic conductivity characteristics of the poroelastic medium are modelled. Investigations of this behaviour suggest that the damage-induced alterations of both stiffness and hydraulic conductivity remain sensibly constant from the state achieved during the initial damage. This suggests the applicability of a stationary damage concept where the initial damage-induced alteration of the poroelastic characteristics leads to a poroelastic medium that is both elastically and hydraulically inhomogeneous. The initial inhomogeneous state for the poroelastic medium can be conveniently determined by considering solutions to the associated elasticity problem. The proposed methodology is applied to examine, computationally, certain classical contact problems for a poroelastic medium susceptible to elastic damage.

Keywords: Damage mechanics; Poroelastic damage; Poroelastic contact; Stationary damage; Biot consolidation; Hydraulic conductivity alteration; Indentation problems; Poroelastic Hertz contact; Boussinesq indentation

1. Introduction

The theory of poroelasticity deals with mechanics of porous elastic materials that are saturated with deformable fluids. In its classical form poroelasticity assumes Hooke's law for the elastic behaviour of the porous skeleton and Darcy's law to describe fluid flow through the porous solid. The linear theory proposed by Biot (1941) has been successfully applied to examine time-dependent transient phenomena encountered in a wide range of natural and synthetic materials, including geomaterials and biomaterials (Detournay and Cheng, 1993; Coussy, 1995; Selvadurai, 1996, 2001a; Cheng et al., 1998; Lewis and Schrefler, 1998; Thimus et al., 1998; de Boer, 1999, 2000; Wang, 2000; Auriault et al., 2002). The assumption of linear elastic behaviour and Darcy flow behaviour of the porous skeleton that will remain unaltered during of loading

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doi:10.1016/j.ijsolstr.2003.08.023

of a poroelastic material is recognized as a limitation of the classical theory. A natural extension to the classical theory is to introduce concepts such as elasto-plasticity to account for irreversible effects in the behaviour of the porous skeleton. Such extensions, however, do not accurately model materials that maintain their poroelastic character but give rise to alterations in the elasticity and fluid flow behaviour as a result of micro-mechanical processes resulting in the generation of micro-cracks and micro-voids in the porous fabric.

One of the basic concepts advocated in this paper rests on the assumption that damage evolution in a poroelastic material leads to an alteration in its fluid transport characteristics and that such alterations can be substantial even for damage levels well below those required to initiate complete failure of the material. The evolution of micro-cracks and micro-voids at stress levels well below the failure load are assumed to be primary mechanisms that could lead to such alterations. The experimental basis for this assumption is provided in a number of investigations, albeit without the advantages of a direct relationship to some measure of damage. For example, experimental investigations by Cook (1965), Bieniawski et al. (1967) and Paterson (1978) point to the load-induced degradation of elastic moduli of rocks. Further investigations along these lines relating to sandstone are discussed by Cheng and Dusseault (1993). Other studies related to the Lac du Bonnet granite are given by Martin and Chandler (1994) and experiments indicating damage effects related to quartzite rock are discussed by Chau and Wong (1997). Similar phenomena have also been observed by Spooner and Dougill (1975), Suhawardy and Pecknold (1978), Mazars and Pijaudier-Cabot (1989a,b) and Bazant and Planas (1998) in connection with experiments conducted on concrete. Topics of related interest are also discussed in the volumes by Elfgren (1989) and Shah and Swartz (1989). The effect of micro-crack generation and damage on the evolution of hydraulic conductivity of saturated geomaterials has been documented by Zoback and Byerlee (1975), who present results of tests conducted on granite that indicate increases of up to a factor of four in the magnitude of the permeability. The results of tests conducted by Shiping et al. (1994) on sandstone indicate that for all combinations of stress states employed in their tests, the permeability increased by an order of magnitude. Kiyama et al. (1996) reported the results of triaxial tests on anisotropic granite, which also indicate increases in the permeability characteristics. Coste et al. (2001) discuss the results of experiments involving rocks and clay stone and their conclusions support the assumption of an increase in the permeability, of up to two-orders of magnitude, with an increase in deviator stresses. Investigations of hydraulic conductivity alterations in excavation damage increase zones have also been reported by Zhang and Dusseault (1997), who used simple constant head borehole tests to evaluate such alterations. Souley et al. (2001) describe the excavation damage-induced alterations in the permeability of granite of the Canadian Shield, where an approximate four-orders of magnitude increase in the permeability in the excavation damage zone is observed. It should be noted, however, that some of these measurements are applicable to stress states where there can be substantial deviations from the elastic response of the material as a result of generation of localized shear zones and foliation type extensive brittle fracture. Clearly, this particular response, although important from the point of view of the modelling of the material for the wider class of stress states, does not address the phenomena related to distributed micro-crack evolution that would permit the application of an elastic model. Experimental results presented by Samaha and Hover (1992) indicate an increase in the permeability of concrete subjected to compression. The study by Gawin et al. (2002) deals with the thermo-mechanical damage of concrete at high temperatures. Here, empirical relationships have been proposed to describe the alterations in the fluid transport characteristics as a function of temperature and damage; the dominant agency responsible for the alterations being thermally-induced generation of micro-cracks and fissures (see also, Schneider and Herbst, 1989; Bary, 1996). Bary et al. (2000) also present experimental results concerning the evolution of permeability of concrete subjected to axial stresses, in connection with the modelling of concrete gravity dams that are subjected to fluid pressures. Results for gas permeability evolution in natural salt during deformation are given by Stormont and Daemen (1992), Schulze et al. (2001), and Popp et al. (2001). These studies support the general trend of permeability increase with damage evolution and the resulting increase in the porosity. The article by Popp et al. (2001) also contains a comprehensive review of further experimental studies in the area of gas permeability evolution in salt during hydrostatic compaction and triaxial deformation. It is, however, recognized that natural salt is a highly creep-susceptible material and the elastic damage is only a minor component of the overall mechanical response. Similar time-dependent effects can also occur in brittle materials due to sub-critical propagation of micro-cracks due to phenomena known as *stress corrosion* (Shao et al., 1997). It should also be remarked that not all stress states contribute to such increases in the permeability of geomaterials. The work of Brace et al. (1978) and Gangi (1978) indicates that the permeability of granitic material can indeed *decrease* with an increase in confining stresses. Similar conclusions are reached by Patsouls and Gripps (1982) in connection with permeability reduction in chalk with the increase in the confining stresses. Wang and Park (2002) also discuss the process of fluid permeability reduction in sedimentary rocks and coal in relation to stress levels. A further set of experimental investigations, notably those by Li et al. (1994, 1997) and Zhu and Wong (1997) also point to the increase in permeability with an increase in the deviator stress levels; these investigations, however, concentrate on the behaviour of the geomaterial in the post peak and, more often on the strain softening range. Again, these experimental investigations, although of considerable interest in their own right, are not within the scope of the current paper that primarily deals with permeability evolution during damage in the elastic range.

Although limited in scope, the available experimental results generally suggest the possibility of extending the classical theory of poroelasticity to include the effects of micro-crack and micro-void generation through consideration of continuum damage modelling. Using a damage mechanics approach, the influences of the deterioration of the elasticity characteristics and enhancement of the hydraulic conductivity characteristics in the porous medium are accounted for in a phenomenological sense. Such a theory is considered to be suitable for describing the mechanical behaviour of brittle elastic solids well in advance of the development of macro-cracks (e.g. fractures) or other irreversible phenomena (e.g. plasticity effects). It should be observed that the poroelastic material itself has a pore space; the damage-induced micro-voids and micro-cracks are assumed to be sparse and substantially larger than the characteristic dimension associated with the pore space in the undamaged material. Admittedly, the damage process is expected to be highly anisotropic in nature and could conceivably be restricted to localized zones. In this study, however, damage is a phenomenological process, resulting from the reduction in the elastic stiffness and enhancement of the fluid conductivity characteristics due to generation of micro-voids and other micro-defects. Cheng and Dusseault (1993) developed an anisotropic damage model to examine the poroelastic behaviour of saturated geomaterials, in the absence of hydraulic conductivity alteration during the damage process. Mahyari and Selvadurai (1998) considered a computational modelling of an axisymmetric poro-elastic contact problem where both elastic stiffness reduction and hydraulic conductivity alteration during damage evolution are considered. Through consideration of pertinent experimental literature, Mahyari and Selvadurai also presented plausible theoretical relationships to describe the damage-induced alterations in the hydraulic conductivity of brittle poroelastic rocks. Recently, Selvadurai and Shirazi (2002) and Shirazi and Selvadurai (2002) utilized the procedures proposed by Mahyari and Selvadurai (1998) to examine the axisymmetric problems related, respectively, to spherical fluid inclusions embedded in a damage-susceptible poroelastic infinite space and the problem of the indentation of a poroelastic halfspace by a rigid cylindrical punch with a smooth flat base. The work of Bary et al. (2000) also examines the coupling effects of damage on hydro-fracturing of concrete and the studies by Gawin et al. (1999, 2002) investigate the influence of thermal effects and the coupling effect to permeability alterations, with specific reference to high temperature loading of partially-saturated concrete. Of related interest are the permeability alterations that take place due to the combined chemical and mechanical action in porous media. A discussion of some recent research in this area is given by Lichtner et al. (1996). The computational studies by Bai et al. (1997), Aublive-Conil et al. (2002), Klubertanz et al. (2002) and Tang et al. (2002) also focus on the topic of permeability-damage coupling at various stages of elastic and elasto-plastic damage evolution. It should be remarked that, although there is extensive literature dealing with the experimental modelling of geomaterial behaviour by appeal to continuum damage mechanics (Aubertin et al., 1998), investigations that deal with the study of the evolution of hydraulic conductivity with damage are relatively limited and to the author's knowledge restricted to the works mentioned previously.

The objective of this paper is to introduce a simpler computational procedure that can be used for the modelling of poroelastic media susceptible to damage. As observed by Mahyari and Selvadurai (1998), the extent of the damage region occurring at the initial loading state remains relatively *stationary* throughout the ensuing transient poroelastic processes. This suggests the possibility of introducing the concept of "stationary damage" as a criterion for the modelling of the poroelasticity problem associated with a material with a porous skeleton that is susceptible to damage. The primary advantage of the *stationary damage* concept is that the resulting poroelasticity problem involves the analysis of a conventional problem in poroelasticity but with a spatial non-homogeneity in the elastic and fluid conductivity properties. The methodology also has further advantages. First, the concept eliminates the need for an iterative analysis that is required to continually update the spatial and temporal evolution of elasticity and hydraulic conductivity values as damage evolves. Second, the deformability and hydraulic conductivity parameters consistent with a state of stationary damage can be conveniently estimated through consideration of appropriate analytical solutions available for an elasticity problem, and finally, the poroelasticity problem for stationary damage is classical and computational procedures applicable to the solution of the problem are well-established.

In this study we specifically consider the application of the stationary damage concept to the analysis of the frictionless indentation of a poroelastic halfspace, separately, by a rigid circular porous indenter with a flat base and a spherical rigid porous indenter. These are two celebrated problems in contact mechanics; their elastic solutions were first considered in the classic studies by Hertz (1882), Boussinesq (1885) and Harding and Sneddon (1945). Computational techniques are applied to examine the influence of elastic damage-induced fluid transport characteristics on the time-dependent indentational response of the two indenters.

2. Equations of poroelasticity

The constitutive equations governing the quasi-static response of a poroelastic medium consisting of a porous isotropic soil skeleton saturated with a compressible pore fluid, take the forms

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{\varepsilon} + \frac{2\mu v}{(1-2v)}(\nabla \cdot \mathbf{u})\mathbf{I} + \alpha p\mathbf{I}, \tag{1}$$

$$p = \beta\zeta_{\mathrm{v}} + \alpha\beta(\nabla \cdot \mathbf{u}), \tag{2}$$

where $\boldsymbol{\sigma}$ is the total stress dyadic, p is the pore fluid pressure, ζ_{v} is the volumetric strain in the compressible pore fluid; v and μ are the "drained values" of Poisson's ratio and the linear elastic shear modulus applicable to the porous fabric, $\mathbf{I}(= \mathbf{ii} + \mathbf{jj} + \mathbf{kk})$ is the unit dyadic, $\boldsymbol{\varepsilon}$ is the soil skeletal strain dyadic and $\mathbf{u}$ is the displacement vector. The material parameters α and β define, respectively, the compressibility of the pore fluid and the compressibility of the soil fabric and are given by

$$\alpha = \frac{3(v_{\mathrm{u}} - v)}{\widetilde{B}(1-2v)(1+v_{\mathrm{u}})}; \quad \beta = \frac{2\mu(1-2v)(1+v_{\mathrm{u}})^2}{9(v_{\mathrm{u}} - v)(1-2v_{\mathrm{u}})}, \tag{3}$$

where v_{u} is the undrained Poisson's ratio and $\widetilde{B}$ is the pore pressure parameter introduced by Skempton (1954). The effective stress dyadic $\boldsymbol{\sigma}'$ of the porous skeleton is given by

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \alpha p\mathbf{I}. \tag{4}$$

In the absence of body forces, the quasi-static equations of equilibrium for the complete fluid saturated porous medium take the form

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0}. \tag{5}$$

The velocity of fluid transport within the pores of the medium is governed by Darcy's law,

$$\mathbf{v} = -\kappa \nabla p, \tag{6}$$

where $\mathbf{v}$ is the vector of fluid velocity and $\kappa\ (= k/\gamma_{\mathrm{w}})$ is a permeability parameter, which is related to the hydraulic conductivity k and the unit weight of the pore fluid γ_{w} as indicated above.

The mass conservation equation for the pore fluid can be stated as an equation of continuity, which, for quasi-static flows can be stated in the form

$$\frac{\partial \zeta_{\mathrm{v}}}{\partial t} + \nabla \cdot \mathbf{v} = 0. \tag{7}$$

Considering the thermodynamic requirements for a positive definite strain energy potential (see e.g. Rice and Cleary, 1976) it can be shown that the material parameters should satisfy the following constraints:

$$\mu > 0; \quad 0 \leqslant \widetilde{B} \leqslant 1; \quad -1 < v < v_{\mathrm{u}} \leqslant 0.5; \quad \kappa > 0. \tag{8}$$

The governing equations for a poroelastic medium can be reduced to the following:

$$\mu \nabla^2 \mathbf{u} + \frac{\mu}{(1-2v)} \nabla(\nabla \cdot \mathbf{u}) + \alpha \nabla p = \mathbf{0} \tag{9}$$

and

$$\kappa \beta \nabla^2 p - \frac{\partial p}{\partial t} + \alpha \beta \frac{\partial}{\partial t}(\nabla \cdot \mathbf{u}) = 0 \tag{10}$$

for the displacement vector $\mathbf{u}$, p is the scalar pore fluid pressure and ∇^2 is Laplace's operator. The above equations are quite general, in the sense that appropriate reductions to the behaviour of a poroelastic material saturated with an incompressible fluid can be recovered from these expressions.

To complete the description of the initial boundary value problem, it is necessary to prescribe boundary conditions and initial conditions for the dependent variables. The boundary conditions can be identified in terms of the Dirichlet, Neumann and Robin-type classical relationships. Also, uniqueness theorems for the initial boundary value problem in classical poroelasticity are available in the literature (Coussy, 1995; Altay and Dokmeci, 1998).

3. Finite element formulations

The finite element modelling of problems in poroelasticity is now well established. The earliest of these studies is due to Sandhu and Wilson (1969) with other noteworthy studies by Ghaboussi and Wilson (1973), Booker and Small (1975), Verruijt (1977), Huyakorn and Pinder (1983), Valliappan et al. (1974) and Simon et al. (1986). Complete discussions of the computational aspects of poroelasticity are also given by Desai and Christian (1977) and Lewis and Schrefler (1998). The Galerkin approximation technique is applied to transform the partial differential Eqs. (9) and (10) into a discretized matrix form. The approximations used for the displacements $\mathbf{u}$ and the pore pressure p can be obtained through

$$\mathbf{u} = N^u\{\mathbf{u}\}; \quad p = N^p\{\tilde{p}\}, \tag{11}$$

where $\{\mathbf{u}\}$ and $\{\tilde{p}\}$ are the nodal displacements and pore pressure vectors and N^u and N^p correspond to the nodal shape functions for the displacement and pore pressure fields, respectively. In general N^u and N^p can be different but both must satisfy C^0 continuity. The application of the Galerkin procedure to the governing equations gives rise to the following incremental forms for the equations governing poroelastic media:

$$\begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^T & \{-\gamma\Delta t\mathbf{H}+\mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{t+\Delta t} \\ \tilde{p}_{t+\Delta t} \end{Bmatrix} = \begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^T & \{(1-\gamma)\Delta t\mathbf{H}+\mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_t \\ \tilde{p}_t \end{Bmatrix} + \{\mathbf{F}\}, \tag{12}$$

where $\mathbf{K}$ is the stiffness matrix of the soil skeleton; $\mathbf{C}$ is the stiffness matrix due to interaction between the soil skeleton and the pore fluid; $\mathbf{E}$ is the compressibility matrix of the pore fluid; $\mathbf{H}$ is the permeability matrix; $\mathbf{F}$ are force vectors due to external tractions, body forces and flows; $\mathbf{u}_t$ and $\tilde{p}_t$ are, respectively, the nodal displacements and pore pressure at time t; and Δt is the time increment. The time integration constant γ varies between 0 and 1. The criteria governing stability of the integration scheme given by Booker and Small (1975) requires that $\gamma \geqslant 1/2$. According to Selvadurai and Nguyen (1995) and Lewis and Schrefler (1998), the stability of the solution can be achieved by selecting values of γ close to unity (see also Wang et al., 2000). In the ensuing we shall apply the classical theory of poroelasticity to the specific contact problems identified in the introduction.

4. Continuum damage mechanics

The development of continuum damage mechanics is generally attributed to Kachanov (1958), who used the concept to describe the mechanics of tertiary creep in solids. Since these initial developments, the theory of continuum damage mechanics has been widely used to predict the mechanical response of a variety of materials including metals, concrete, composites, frozen soil and other geological materials (see e.g. Bazant, 1986; Simo and Ju, 1987; Costin, 1989; Mazars and Bazant, 1989; Lemaitre and Chaboche, 1990; Selvadurai and Hu, 1995; Krajcinovic, 1996; Mahyari and Selvadurai, 1998; Voyiadjis et al., 1998; Ju, 1990). In the ensuing we shall present, for completeness, a brief outline of the essential features of the phenomenological approach to continuum damage mechanics; more comprehensive discussions can be found in the references cited previously. It should also be noted, that with the use of the concept of stationary damage proposed here, it is assumed at the outset that the damage-induced alterations in the elasticity properties of the fluid-saturated imedium particularly in the initial stages of the poroelastic process are less important than the significant alterations in the hydraulic conductivity characteristics. Therefore, a fluid-saturated poroelastic medium that experiences stationary damage can also be conveniently modelled as a hydraulically inhomogeneous medium where such inhomogeneities are derived through considerations of a solution to an associated problem in elasticity. The alterations in the hydraulic conductivity characteristics can be related to either a stress or a strain invariant, which could also be related to the damage variable used in the description of conventional scalar isotropic damage mechanics. In a general sense, however, the evolution of *both* elastic inhomogeneity and hydraulic inhomogeneity can be accommodated in the stationary damage modelling.

The non-linear elastic damage development in most brittle materials is generally attributed to the initiation of new micro-defects and the growth of existing micro-defects. This behaviour can be modelled by introducing local continuous damage variables in the analysis. In cases where the damage process results in micro-voids with a spherical form or micro-cracks that have a random size distribution and orientation, the damage phenomena can be described by appeal to the scalar damage variable D (Kachanov, 1958; Mazars and Pijaudier-Cabot, 1989a; Krajcinovic, 1996) given by

$$D = \frac{(A_0 - \overline{A})}{A_0}, \tag{13}$$

where A_0 is the initial area and $\overline{A}$ is the reduced net area. The damage variable varies between 0 and D_c, where D_c is a critical value of damage, corresponding to the fracture of the material. (The critical damage parameter can be viewed as a normalizing parameter against which other levels of damage evolution can be measured.) This definition facilitates the adaptation of the damage concept within the theory of elasticity or in any other theory associated with classical continuum mechanics. The coupling of elasticity with damage models has been investigated by a number of researchers and references to these studies can be found in the works by Sidoroff (1980), Mazars (1982), Chow and Wang (1987), Mazars and Pijaudier-Cabot (1989a,b), Lemaitre and Chaboche (1990), Lemaitre (1992), Krajcinovic (1996), Swoboda et al. (1998) and Eskandari and Nemes (1999). In keeping with the approach proposed by Mazars and Pijaudier-Cabot (1989a), the damage is assumed to be elastic, scalar isotropic and only the elastic parameters are assumed to be altered by damage. The introduction of the damage variable D leads directly to the concept of a net stress defined in relation to the net area available for load transfer. For isotropic damage, the net stress dyadic $\boldsymbol{\sigma}^n$ is related to the stress dyadic in the undamaged state $\boldsymbol{\sigma}$ according to

$$\boldsymbol{\sigma}^n = \frac{\boldsymbol{\sigma}}{(1-D)}. \tag{14}$$

Considering the 'strain equivalence hypothesis' proposed by Lemaitre (1984) and Lemaitre (1992), the constitutive equation for the damaged skeleton of the poroelastic medium can be written as

$$\boldsymbol{\sigma} = 2(1-D)\mu\boldsymbol{\varepsilon} + \frac{2(1-D)\mu v}{(1-2v)}(\nabla\cdot\mathbf{u})\mathbf{I} + \alpha p\mathbf{I} \tag{15}$$

which implies that Poisson's ratio remains constant, which is an added constraint when considering the material behaviour in three-dimensions. We further note that although there is a reduction in the area available for the transfer of the geomaterial skeletal stresses, in (15) we have neglected the corresponding increase in area available for the transmission of the pore fluid pressures. Also, in the extension of the constitutive response from (1)–(15) to take into consideration the influence of damage, we have excluded any dependency of the damage state on either the sense of the stress state or a damage-based loading surface. Such treatments are available in the literature (Mazars and Pijaudier-Cabot, 1989b; Carol et al., 1994) but in the interests of focusing attention on the stationary damage concept, such investigations are relegated to further studies.

In addition to the specification of the constitutive relations for the damaged geomaterial skeleton, it is also necessary to prescribe damage evolution criteria that can be postulated either by appeal to micromechanical considerations or determined by experiment. For example, based on a review of the results of experiments conducted on rocks it has been shown (Cheng and Dusseault, 1993; Cheng et al., 1993) that

$$\frac{\partial D}{\partial \xi_d} = \eta \frac{\gamma \xi_d}{(1+\xi_d)}\left(1 - \frac{D}{D_c}\right), \tag{16}$$

where ξ_d (the equivalent shear strain) is related to the second invariant of the deviator strain dyadic and η and γ are positive material constants. Also

$$\xi_d = \operatorname{tr}\mathbf{e}^2; \quad \mathbf{e} = \boldsymbol{\varepsilon} - \frac{1}{3}\operatorname{tr}\boldsymbol{\varepsilon}\mathbf{I}. \tag{17}$$

In this formulation, the normalizing damage measure is the critical damage D_c, which is associated with the damage corresponding to a residual value of the strength of the geomaterial under uni-axial compression. This is not a restriction; the normalizing value could equally well be taken as the damage measure at the attainment of the peak load. The evolution of the damage variable can be obtained by the integration of (16) between the limits D_0 and D, where D_0 is the initial value of the damage variable corresponding to the intact state. (e.g. D_0 is zero for materials in a virgin state.) The deformability parameters applicable to an initially isotropic elastic material that experiences isotropic damage can be updated by adjusting the elastic constants in the elasticity matrix $\mathbf{C}$ by its equivalent applicable to the damaged state $\mathbf{C}^d$ but maintaining Poisson's ratio constant, such that the elasticity matrix applicable to damaged materials is given as

$$\mathbf{C}^{\mathrm{d}} = (1 - D)\mathbf{C}. \tag{18}$$

Integrating (16) between appropriate limits, the evolution of D can be prescribed as follows:

$$D = D_{\mathrm{c}} - (D_{\mathrm{c}} - D_0)(1 + \gamma\xi_{\mathrm{d}})^{\eta/\gamma D_{\mathrm{c}}}.\exp(-\eta\xi_{\mathrm{d}}/D_{\mathrm{c}}) \tag{19}$$

The development of damage-related criteria that can account for alterations in the hydraulic conductivity during the evolution of mechanical damage in saturated geomaterials is a necessary component for the computational modelling. As discussed previously, literature on the coupling between micro-crack development and permeability evolution in saturated geomaterials is primarily restricted to the experimental evaluation of alteration in the permeability of geomaterials, which are subjected to a triaxial stress state. Zoback and Byerlee (1975) have documented results of experiments conducted on granite and Shiping et al. (1994) give similar results for tests conducted on sandstone. They have observed that the permeability characteristics of these materials can increase by an order of magnitude before the attainment of the peak values of stress and they can increase up to two-orders of magnitude in the strain softening regime where micro-cracks tend to localize in shear faults. Kiyama et al. (1996) also observed similar results for the permeability evolution of granites subjected to a triaxial stress state. This would suggest that localization phenomena could result in significant changes in the permeability in the localization zones. It must be emphasized that in this study the process of localization is excluded from the analysis and all changes in permeability are at stress states well below those necessary to initiate localization or global failure of the material. Furthermore, in keeping with the approximation concerning scalar isotropic dependency of the elasticity properties on the damage parameter, we shall assume that the alterations in the permeability characteristics also follow an isotropic form. This is clearly an approximation with reference to the mechanical response of brittle geomaterials that tend to develop micro-cracking along the dominant direction of stressing, leading to higher permeabilities in orthogonal directions. The studies by Mahyari and Selvadurai (1998) suggest the following postulates for the evolution of hydraulic conductivity as a function of the parameter ξ_{d} as follows:

$$k^{\mathrm{d}} = (1 + \alpha\xi_{\mathrm{d}})k; \quad k^{\mathrm{d}} = (1 + \beta\xi_{\mathrm{d}}^2)k, \tag{20}$$

where k^{d} is the hydraulic conductivity applicable to damaged material, k is the hydraulic conductivity of the undamaged material and α, β are material constants. In computational procedures that *do not* invoke the stationary damage concept, damage evolution criteria governing the time- and position-dependent alteration of elastic stiffness and hydraulic conductivity can be accounted for.

5. Indentation problems

Indentation and contact problems occupy an important position in both engineering and applied mechanics. Solutions derived for classical elastostatic contact problems have been applied to examine the mechanics of indenters used for materials testing, mechanics of nano-indenters, tribology, mechanics of foundations used for structural support, biomechanical applications for prosthetic implants and more recently in the area of contact mechanics of electronic storage devices. Detailed accounts of mathematical developments associated with contact mechanics can be found in a number of specialized volumes and articles devoted to the subject including the works of Galin (1961), Lur'e (1964), Ufliand (1965), Ling (1973), Goodman (1974), de Pater and Kalker (1975), Selvadurai (1979, 2000a), Gladwell (1980), Johnson (1985), Curnier (1992), Kalker (1990), Barber and Ciavarella (2000), Bushan (1998), Fischer-Cripps (2000, 2002), Aleynikov (2000), Alexandrov and Pozharskii (2001) and Willner (2003).

We first consider the problem of the *frictionless indentation* of a poroelastic material by a rigid circular punch with a flat base (Fig. 1a), which is subjected to a total load P_0. The associated classical elasticity

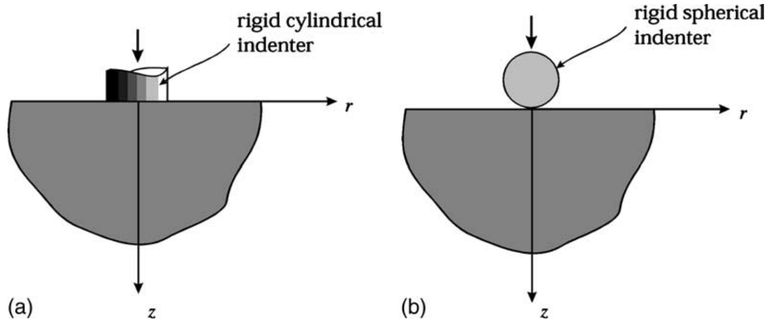

Fig. 1. Indentation of a damage susceptible poroelastic halfspace.

solution was first given by Boussinesq (1885), who examined the problem by considering the equivalence between the elastostatic problem and the equivalent problem in potential theory. Harding and Sneddon (1945) subsequently examined the problem in the classic paper that uses Hankel transform techniques to reduce the problem to the solution of a system of dual integral equations. The procedure also resulted in the evaluation of the load–displacement relationship for the indenter in exact closed form. In a subsequent paper, Sneddon (1946) presented complete numerical results for the distribution of stresses within the halfspace region. The classical poroelasticity problem concerning the static indentation of a poroelastic halfspace and layer regions by a rigid circular indenter with a flat smooth base and related contact problems were considered by a number of authors including Agbezuge and Deresiewicz (1974), Chiarella and Booker (1975), Szefer and Gaszynski (1975), Gaszynski (1976), Gaszynski and Szefer (1978), Selvadurai and Yue (1994), Yue and Selvadurai (1994, 1995a,b, 1996) and Lan and Selvadurai (1996), using differing boundary conditions related to the pore pressure at the surface of the halfspace both within the contact zone and exterior to it. These authors also use different computational schemes for the numerical solution of the resulting integral equations and for the inversion of Laplace transforms. Of related interest are problems associated with the dynamic problem of a rigid foundation either in smooth contact or bonded to the surface of a halfspace (Halpern and Christiano, 1986; Kassir and Xu, 1988; Philippacopoulos, 1989; Bougacha et al., 1993; Senjuntichai and Rajapakse, 1996), where, in certain circumstances, the static transient poroelasticity solution can be recovered. The former studies will form a basis for a comparison with the modelling involving stationary damage; in the present paper, this analysis will consider *only* changes in the hydraulic conductivity characteristics, which will be altered corresponding to the initial elastic strains induced during the loading of the indenter. Also, the load applied is specified in the form of a Heaviside step function in time. In order to determine the stationary spatial variation of hydraulic conductivity properties within the halfspace region, it is first necessary to determine the distribution of the equivalent shear strain ξ_{d} in the halfspace region. Formally, the distribution $\xi_{\mathrm{d}}(r,z)$, can be determined by considering the stress state in the halfspace region associated with the elastic contact stress distribution at the indenter-elastic halfspace region, which is given by

$$\sigma_{zz}(r,0) = \frac{P_0}{2\pi a\sqrt{a^2 - r^2}}; \quad r \in (0,a) \tag{21}$$

and the classical solution by Boussinesq (1885) for the problem for the action of a concentrated normal load at the surface of a halfspace region (see also Selvadurai, 2000a, 2001b). The displacement distribution at the surface of the halfspace region is given by

$$u_z(r,0) = \begin{cases} \Delta; & r \in (0,a), \\ \frac{2\Delta}{\pi}\sin^{-1}\left(\frac{a}{r}\right); & r \in (a,\infty). \end{cases} \tag{22}$$

The stress state in the halfspace region is given by

$$\begin{aligned}
\sigma_{rr}(\rho,\zeta) &= -\frac{P_0}{2\pi a^2}\left[J_1^0 + 2\tilde{v}\{J_1^0 - J_0^1\} - \zeta J_2^0 - \frac{1}{\rho}\{(1-2\tilde{v})J_0^1 - \zeta J_2^1\}\right],\\
\sigma_{\theta\theta}(\rho,\zeta) &= -\frac{P_0}{2\pi a^2}\left[2\tilde{v}J_0^1 + \frac{1}{\rho}\left\{(1-2\tilde{v})J_0^1 - \zeta J_2^1\right\}\right],\\
\sigma_{zz}(\rho,\zeta) &= -\frac{P_0}{2\pi a^2}[J_1^0 + \zeta J_2^0],\\
\sigma_{rz}(\rho,\zeta) &= -\frac{P_0}{2\pi a^2}[\zeta J_2^1],
\end{aligned} \tag{23}$$

where $\tilde{v}$ is Poisson's ratio for the elastic solid and the infinite integrals $J_n^m(\rho,\zeta)$ are defined by

$$J_n^m(\rho,\zeta) = \int_0^\infty s^{n-1}\sin(s)\exp(-s\zeta)J_m(s\rho)\,\mathrm{d}s. \tag{24}$$

As has been shown by Sneddon (1946), these infinite integrals can be evaluated in explicit closed form as follows:

$$\begin{aligned}
J_1^0(\rho,\zeta) &= \frac{1}{\sqrt{R}}\sin\left(\frac{\phi}{2}\right); \quad J_0^1(\rho,\zeta) = \frac{1}{\rho}\left(1 - \sqrt{R}\sin\left(\frac{\phi}{2}\right)\right),\\
J_1^1(\rho,\zeta) &= \frac{\Psi}{\rho\sqrt{R}}\sin\left(\theta - \frac{\phi}{2}\right); \quad J_2^1(\rho,\zeta) = \frac{\rho}{R^{3/2}}\sin\left(\frac{3\phi}{2}\right),\\
J_2^0(\rho,\zeta) &= \frac{\Psi}{R^{3/2}}\sin\frac{3}{2}(\phi - \theta),
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
&\tan\theta = \frac{1}{\zeta}; \quad \tan\phi = \frac{2\zeta}{(\rho^2 + \zeta^2 - 1)}; \quad \Psi^2 = (1 + \zeta^2),\\
&R^2 = [(\rho^2 + \zeta^2 - 1)^2 + 4\zeta^2]; \quad \rho = \frac{r}{a}; \quad \zeta = \frac{z}{a}.
\end{aligned} \tag{26}$$

The principal stress components are determined from the relationships

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{1}{2}\left[(\sigma_{rr} + \sigma_{zz}) \pm \sqrt{(\sigma_{rr} - \sigma_{zz})^2 + 4\sigma_{rz}^2}\right]; \quad \sigma_2 = \sigma_{\theta\theta} \tag{27}$$

and the equivalent shear strain ξ_d can be expressed in the form

$$\xi_\mathrm{d} = \frac{1}{2\sqrt{3}\tilde{\mu}}[(\sigma_1 - \sigma_3)^2 + (\sigma_3 - \sigma_2)^2 + (\sigma_2 - \sigma_1)^2]^{1/2}, \tag{28}$$

where $\tilde{\mu}$ is the linear elastic shear modulus for the elastic solid. In these general elasticity solutions the elastic constants $\tilde{\mu}$ and $\tilde{v}$ can be assigned the values corresponding to their values at time $t = 0$, to reflect the undrained behaviour of the poroelastic solid.

An inspection of both (21) and (23) indicates that the elastic stress state is singular at the boundary of the rigid indenter. This is, of course, a restriction on the rigorous application of the stress state (23) for the determination of damaged regions. Such regions, in principle, are assumed to experience only finite levels of isotropic damage, which essentially maintain the elastic character of the material. In reality, the occurrence of the singular stress state will initiate either plastic failure of the material (Ling, 1973; Johnson, 1985) or even brittle fracture extension in the halfspace region (Selvadurai, 2000b). Such manifestations are assumed to be restricted to a very limited zone of the halfspace region in the vicinity of the boundary of the indenter region. Particularly in view of the computational modelling of the contact problem, there is no provision for the incorporation of singularity elements at the boundary of the contact zone to account for modelling the singular stress state that can be identified from mathematical considerations of the contact problem. In the computational modelling, the mesh configuration is suitably refined to account for the sharp stress gradients that will result from the elastic stress state (Fig. 2). The distribution of equivalent shear strain is accounted for by assigning the values of the equivalent strains to the integration points within the elements. These in turn are converted to alterations in the hydraulic conductivity characteristics of the medium through the use of the expressions (20) that relate the hydraulic conductivity to the equivalent shear strain. The computational modelling is performed using the general purpose finite element code ABAQUS, although any computational code that is capable of examining poroelasticity problems for inhomogeneous media can be adopted for the purpose. An 8-noded isoparametric finite element is used in the modelling and the integrations are performed at the nine Gaussian points. The displacements are specified at all nodes and the pore pressures are specified only at the corner nodes.

The second example deals with the problem of the frictionless indentation of the surface of a poroelastic halfspace susceptible to damage by a rigid porous sphere (Fig. 1b). The associated elasticity problem is the celebrated contact problem of Hertz (1882, 1895). This problem can also be solved by recourse to either methods in potential theory or by appeal a Hankel transform development of the mixed boundary value problem, which results in a set of dual integral equations that can be solved in a standard fashion (see e.g. Sneddon, 1965; Gladwell, 1980; Selvadurai, 2000c). The solution of the elasticity problem gives the contact stresses at the spherical indenter-elastic halfspace interface as

$$u_z(r,0) = \frac{3P_0(1-\tilde{\nu})}{8\tilde{\mu}a}\begin{cases} \left(1-\frac{r^2}{2a^2}\right); & r \in (0,a), \\ \frac{1}{\pi}\left[\left(2-\frac{r^2}{a^2}\right)\sin^{-1}\left(\frac{a}{r}\right)+\frac{r}{a}\left(1-\frac{a^2}{r^2}\right)^{1/2}\right]; & r \in (a,\infty). \end{cases} \tag{29}$$

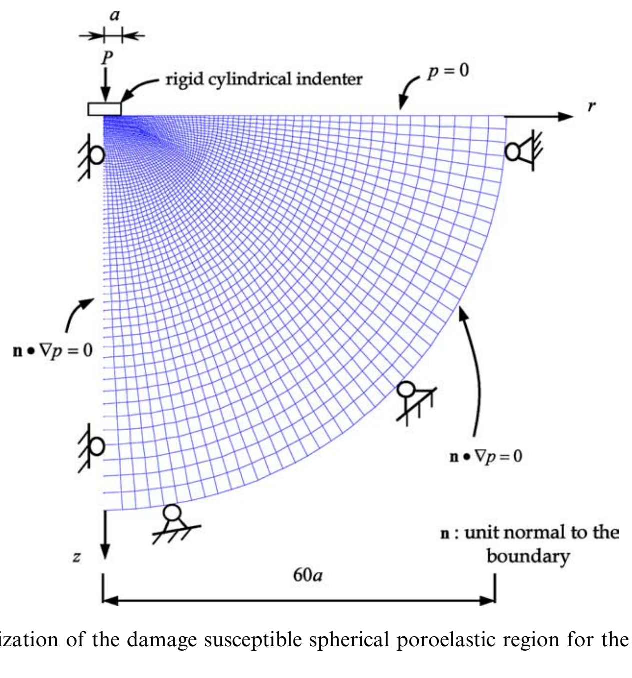

Fig. 2. Finite element discretization of the damage susceptible spherical poroelastic region for the cylindrical indenter problem.

The stress state within the halfspace region can be obtained through the use of Boussinesq's solution and contact normal stress at the sphere-elastic halfspace interface:

$$\sigma_{zz}(r,0) = \frac{3P_0}{2\pi a^2}\left(1-\frac{r^2}{a^2}\right)^{1/2}; \quad r\in(0,a). \tag{30}$$

The stress components are given by the following:

$$\begin{aligned}\sigma_{rr}(\rho,\zeta) = \frac{3P_0}{2\pi a^2}\Bigg\{&\frac{(1-2\tilde{v})}{3\rho^2}\left[1-\left(\frac{\zeta}{\sqrt{\Omega}}\right)^3\right]+\left(\frac{\zeta}{\sqrt{\Omega}}\right)^3\left(\frac{\Omega}{\Omega^2+\zeta^2}\right)\\ &+\frac{\zeta}{\sqrt{\Omega}}\left[\frac{(1-\tilde{v})\Omega}{(1+\Omega)}+(1+\tilde{v})\sqrt{\Omega}\tan^{-1}\left(\frac{1}{\sqrt{\Omega}}\right)-2\right]\Bigg\},\end{aligned}$$

$$\sigma_{\theta\theta}(\rho,\zeta) = -\frac{3P_0}{2\pi a^2}\left\{\frac{(1-2\tilde{v})}{3\rho^2}\left[1-\left(\frac{\zeta}{\sqrt{\Omega}}\right)^3\right]+\frac{\zeta}{\sqrt{\Omega}}\left[2\tilde{v}+\frac{(1-v)\Omega}{(1+\Omega)}-(1+\tilde{v})\sqrt{\Omega}\tan^{-1}\left(\frac{1}{\sqrt{\Omega}}\right)\right]\right\}, \tag{31}$$

$$\sigma_{zz}(\rho,\zeta) = -\frac{3P_0}{2\pi a^2}\left\{\frac{\zeta}{\sqrt{\Omega}}\right\}^3\left\{\frac{\Omega}{\Omega^2+\zeta^2}\right\},$$

$$\sigma_{rz}(\rho,\zeta) = -\frac{3P_0}{2\pi a^2}\left\{\frac{\rho\zeta^2\sqrt{\Omega}}{(1+\Omega)(\Omega^2+\zeta^2)}\right\},$$

where

$$\Omega = [\rho^2+\zeta^2-1+\{[\rho^2+\zeta^2-1]^2+4\zeta^2\}^{1/2}]. \tag{32}$$

The corresponding principal stresses and the principal strains can be obtained from the expressions (26) and the equivalent shear strain can be obtained from the expression (27).

6. Computational results

In this section we consider the computational modelling of the indentation problems associated with a poroelastic solid that can experience stationary damage during the initial indentation process. Attention is restricted to materials that experience only *alterations in the hydraulic conductivity* properties according to the linear relationship defined by (20). For the purpose of these studies, the pore fluid that saturates the porous elastic solid is assumed to be incompressible. The indenters are subjected to a time-dependent load, prescribed as a Heaviside step function i.e.:

$$P(t) = P_0H(t); \quad t>0, \tag{33}$$

where P_0 is the magnitude of the total load acting on the indenter. Since the saturating fluid is assumed to be incompressible, the undrained behaviour of the fluid saturated poroelastic medium at time $t=0$ corresponds to an elastic state with $\tilde{v}=1/2$. The initial elastic strains that induce the spatial distribution of damage during the indentation are evaluated by setting $\tilde{v}=1/2$ in the principal stresses computed, using the stress states (23) and (31), relevant, respectively, to the smooth indentation of the halfspace with the porous rigid circular cylinder with a smooth flat base and the porous spherical indenter with a smooth surface.

The finite element discretization of the halfspace domain used for the analysis of the indentation of the poroelastic halfspace by the porous rigid circular smooth indenter, of radius $a = 2m$, is shown in Fig. 2. Admittedly, to exactly model a poroelastic halfspace region it is necessary to incorporate special infinite elements that capture the spatial decay in the pressure and displacement fields. While the use of infinite element techniques is documented in finite element literature (Simoni and Schrefler, 1987; Selvadurai and Gopal, 1989), these procedures are unavailable in the code that is being used in the current computational modelling. For this computational modelling, the poroelastic region chosen is modelled as a hemispherical domain, where the outer boundary is located at a large distance ($60a$) from the origin (Fig. 2). This external spherical boundary is considered to rigid and all displacements on this boundary are constrained to be zero. It is also considered impervious, thereby imposing Neumann boundary conditions for the pore pressure field at this spherical surface. (It is noted here that computations were also performed by prescribing Dirichlet boundary conditions on this surface. The consolidation responses computed were essentially independent of the far field pore pressure boundary condition.) The accuracy of the discretization procedures, particularly with reference to the location of the external spherical boundary at a distance $60a$, was first verified through comparisons with the Boussinesq's solution for the indentation of an elastic halfspace region by a cylindrical punch. The value of the elastic displacement of the rigid circular indenter can be determined to an accuracy of approximately three percent. The computational modelling of the poroelastic indentation problem is performed by specifying the following values for the material parameters generally applicable to a geomaterial such as sandstone (see e.g. Cheng and Dusseault, 1993): hydraulic conductivity $k = 1 \times 10^{-6}$ m/s; unit weight of pore fluid $\gamma_w = 1 \times 10^4$ N/m^3; Young's modulus $E = 8.3$ GPa; Poisson's ratio $\nu = 0.195$; the corresponding coefficient of consolidation, defined by

$$c = \frac{2\mu k}{\gamma_w} = 0.6946 \text{ m}^2/\text{s}. \quad (34)$$

Fig. 3 illustrates the comparison between the analytical solution for the time-dependent variation in the rigid displacement of the circular indenter given by Chiarella and Booker (1975) and the computational results obtained for the indentation of a poroelastic region with an external boundary in the shape of a hemisphere. It should be noted that the comparison is between an estimate for a halfspace region and a region of finite extent. There is reasonable correlation between the two sets of results. The purpose of the comparison of solutions is to provide a basic estimate of the accuracy of the computational procedure and not to exactly model the deformational behaviour of a poroelastic halfspace region. Since the influence of effects of stationary damage will be assessed in relation to the computational results derived for the indentation of the hemispherical region, the accuracy of the computational scheme in modelling the

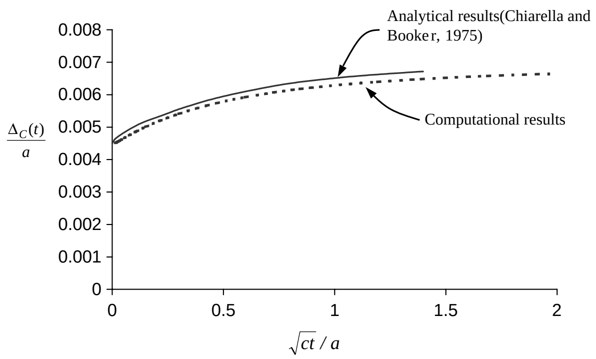

Fig. 3. Comparison of analytical results and computational estimates for the cylindrical rigid indenter problem.

indentation process is considered to be acceptable. The normal stress distribution at the contact zone can be determined to within a similar accuracy except in regions near the boundary of the indenter where, theoretically, the contact stresses are singular. The procedure for examining the poroelasticity problem with stationary damage first involves the determination of the equivalent shear strain distribution within the halfspace region subjected to indentation by the rigid cylinder and defined by (28). In the computational treatments, the stationary damage-induced alteration in the hydraulic conductivity is evaluated according to the *linear dependency* in the hydraulic conductivity alteration relationship given by (20) and the parameter α is varied within the range $\alpha \in (0, 10^4)$. Fig. 4 illustrates the results for the time-dependent displacement of the rigid cylindrical smooth indenter resting on a poroelastic hemispherical domain that displays either stationary damage-induced alteration in the hydraulic conductivity or is independent of such effects. These results are for a specific value of the hydraulic conductivity altering parameter, $\alpha = 10^3$. Computations can also be performed to determine the influence of the parameter α and the stationary damage-induced alterations in the hydraulic conductivity on the settlement rate of the rigid indenter. The results can be best illustrated through the definition of a "Degree of Consolidation ", defined by

$$U_C = \frac{\Delta_C(t) - \Delta_C(0)}{\Delta_C(\infty) - \Delta_C(0)}. \tag{35}$$

The value $\Delta_C(t)$ corresponds to the time-dependent rigid displacement of the indenter. Both the initial and ultimate values of this displacement, for the purely poroelastic case, can be evaluated, independent of the considerations of the transient poroelastic responses since the poroelastic model allows for purely elastic behaviour at $t = 0$ and as $t \to \infty$, with $\nu = 1/2$ and $\tilde{\nu} = \nu$, respectively, i.e.

$$\Delta_C(0) = \frac{P}{8\mu a}; \quad \Delta_C(\infty) = \frac{P(1-\nu)}{4\mu a}. \tag{36}$$

Fig. 5 illustrates the variation of the degree of consolidation of the rigid indenter as a function of the non-dimensional time and the parameter α that defined the alteration of the hydraulic conductivity with the equivalent shear strain ξ_d.

We next consider the problem of the frictionless indentation of the poroelastic halfspace exhibiting stationary damage-induced alteration in the hydraulic conductivity, by a rigid spherical porous indenter. This is a classical advancing contact problem in the theory of elasticity, the solution to which was given by Hertz (1882, 1895). In the case of the elasticity problem, exact closed form solutions can be obtained for the radius of contact, the normal stresses in the Euclidean contact plane and the penetration of the indenter. The stress state in the elastic halfspace can also be obtained in compact closed form. When considering the

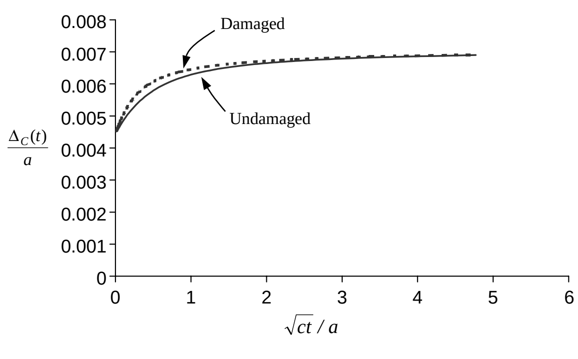

Fig. 4. Influence of stationary damage on the displacement of the rigid cylindrical indenter [$\alpha = 1000$].

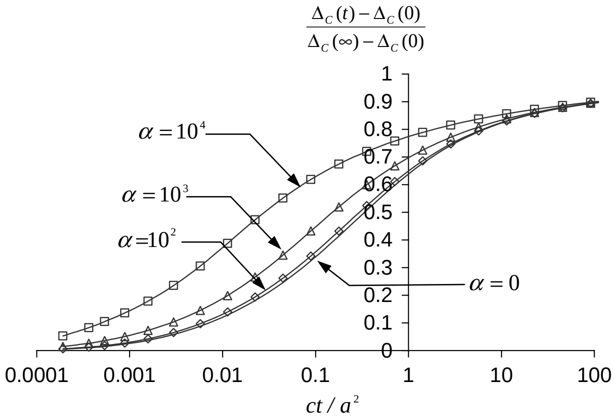

Fig. 5. Influence of stationary damage on the consolidation rate for the rigid cylindrical indenter.

extension to the poroelastic case, the moving boundary nature of the advancing contact problem has to be accounted for on the method of solution, in the sense that the computations have to be conducted in an incremental fashion, identifying the change in the boundary conditions from a traction-specified to a displacement-specified variety, at each time increment. Investigations of certain types of moving boundary problems associated with purely poroelastic contact problems indicate that the contact region maintains a stationary value consistent with the dimensions originally derived through the analysis of the initial elasticity problem (Selvadurai and Mahyari, 1998). Considering Hertz's solution, (i) the initial radius of the circular area of contact a_0, between an elastic halfspace and a rigid smooth sphere of radius R, which is subjected to a force P inducing axisymmetric indentation and (ii) the initial indentation $\Delta_S(0)$, are given by

$$a_0 = \sqrt[3]{\frac{3PR}{16\mu}}; \quad \Delta_S(0) = \frac{a_0^2}{R}. \tag{37}$$

The radius of the contact region at infinite time and the displacement at infinite time are given by

$$a_\infty = \sqrt[3]{2(1-\nu)}a_0; \quad \Delta_S(\infty) = [2(1-\nu)]^{2/3}\Delta_S(0). \tag{38}$$

As is evident, for $\nu \in (0, 1/2)$, the radii ratio $(a_\infty/a_0) \in (1.26, 1)$. Therefore, in this case, the geometry of the contact region does make an appreciable change in its dimension as the consolidation takes place. In the current study, the computational procedure can take into consideration, through a penalty function approach the advancing contact between the rigid porous smooth sphere and the hemispherical poroelastic region exhibiting either no damage or stationary damage. Since the indenting spherical region is considered to be porous, the constraints applicable to an advancing contact are applicable only to the displacements and contact tractions. The contact stress in the Euclidean plane, which will still satisfy the zero-normal stress condition at the point of separation, will be required to determine the unknown radius of the contact zone. Fig. 6 illustrates the finite element discretization used in connection with the computational modelling of the indentation problem related to a rigid porous sphere. The constitutive parameters used for the poroelastic material are identical to those used in connection with the indentation problem for a flat punch. Other parameters are assigned as follows: $P = 5.02 \times 10^8 N$; $R = 2m$. Fig. 7 illustrates a comparison between the time-dependent displacement response for the rigid sphere obtained from the computational modelling with equivalent results given by Agbezuge and Deresiewicz (1974) for the indentation of a

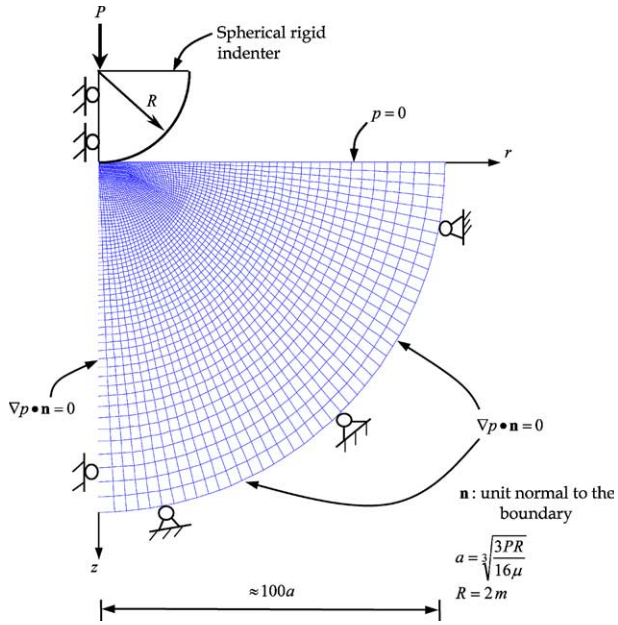

Fig. 6. Finite element discretization of the damage susceptible spherical poroelastic region for the spherical indenter problem.

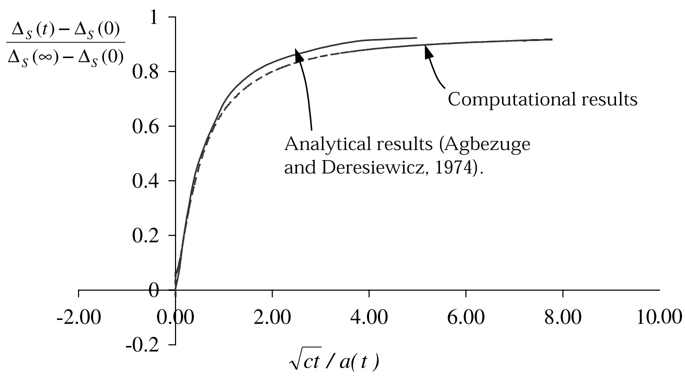

Fig. 7. Comparison of analytical results and computational estimates for the spherical rigid indenter problem.

halfspace region by a rigid, smooth porous sphere. The general trends in the time-dependent behaviour obtained for both solution schemes compare reasonably well. The influence of the permeability alteration during the initial indentation and it influence on the resulting consolidation behaviour of the spherical indenter is shown in Fig. 8. Again, the influence of the permeability alteration due to damage on the results for the indentation by the sphere are best illustrated through the definition of a ‘‘Degree of Consolidation’’, defined by

$$U_S = \frac{\Delta_S(t) - \Delta_S(0)}{\Delta_S(\infty) - \Delta_S(0)}. \quad (39)$$

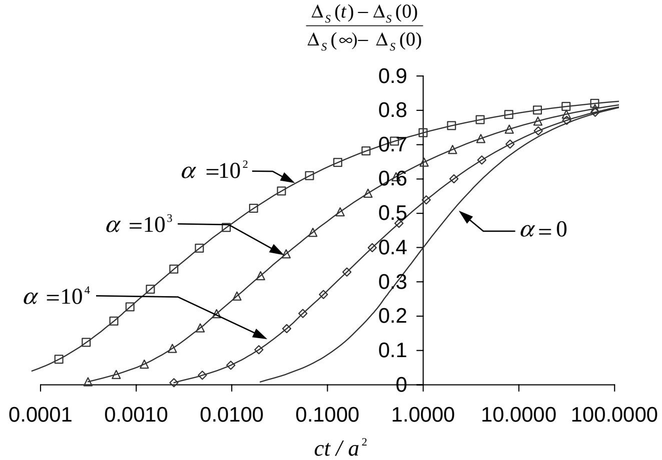

Fig. 8. Influence of stationary damage on the consolidation rate for the rigid spherical indenter [$\alpha = 1000$].

7. Concluding remarks

The classical theory of poroelasticity can be extended to include situations where the elastic damage of the porous skeleton can be accommodated through continuum damage mechanics. In general, such damage can induce alterations in both the elasticity characteristics of the porous skeleton and in the fluid transport or hydraulic conductivity characteristics. While the damage evolution is generally anisotropic, the theory of isotropic damage serves as a useful first approximation for the consideration of such influences. Also, in general, the damage in the poroelastic medium evolves in view of the time-dependent behaviour of the coupled processes. This paper introduces the concept of a stationary damage in the fluid saturated porous medium, which limits the damage evolution to the *initial damage state* resulting from the elastic stress state corresponding to the loading applied to the poroelastic medium. This results in an inhomogeneous poroelastic material with altered elasticity and hydraulic conductivities in the porous domain. Furthermore, existing elastic solutions can be used to considerable advantage to derive the initial stress state, from which the alterations can be deduced, through consideration of relationships derived from plausible experimental results. In particular, the changes in the hydraulic conductivity evolution are considered to be a factor of some importance to the assessment of the rates of consolidation. The stationary damage concept is used to examine, computationally, the time-dependent behaviour of the indentation of a poroelastic region by a rigid indenter with a flat base and a rigid spherical indenter. The results of the computational modelling of a typical poroelastic contact problem indicates that although the damage-induced alteration in the hydraulic conductivity has a significant influence on the time-dependent displacements of the rigid indenters. In the test examples, since there is no provision to take into account alterations in the deformability characteristics of the poroelastic solid, the magnitude of the contact displacements remain relatively unchanged from the classical poroelastic estimates. The computational techniques associated with stationary damage concept can be conveniently extended to include situations where both the elasticity and hydraulic conductivity characteristics of the porous solid can also exhibit a directional and damage stress space-dependency. Such a scheme will still be an *approximate treatment* of a much more complicated computational modelling problem that should address the time-dependent progress of damage evolution within the poroelastic region, but with the additional constraints of damage stress-space and directional dependency and irreversibility in the evolution of its mechanical and hydraulic responses. The stationary damage modelling, however, provides a convenient computational approach for examining, relatively conveniently, the influence of damage-induced hydraulic conductivity alteration on the consolidation rates for poroelastic media.

Acknowledgements

It is indeed a great pleasure for the author to contribute a paper to the Special Issue of the Journal honouring Bruno Boley and his lasting contributions to the development of Applied Mechanics in the Americas. The work described in this paper was initiated during the tenure of a Killam Research Fellowship awarded by the Canada Council for the Arts. The author is grateful for this support and thanks are due to Mr. Qifeng Yu for assistance with the finite element computations.

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